QP CODE: 26023301

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS)SPECIAL RE APPEARANCE EXAMINATIONS, MAY 2026

(EXCLUSIVELY FOR THE FAILED CANDIDATES OF 2023 ADMISSION)

Sixth Semester

CORE COURSE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2023 Admission Only

F241F637

Time: 3 Hours Max. Marks : 80

core

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define a harmonic function.Give an example.
2. Verify the analiticity of f(z)=sinz using CR Equations.
3. Prove that Log $(1+i)^2$= 2Log (1+i),but Log $(-1+i)^2 \neq$ 2Log (-1+i)
4. Define the hyperbolic sine and hyperbolic cosine of a complex variable z
5. Evaluate $\cosh^{-1}(-1)$
6. Evaluate $\int_1^2 (\frac{1}{t} - i)^2 dt.$
7. If C is any simple closed contour, then evaluate $\int_C exp(z^3)dz.$
8. Define simply connected and multiply connected domain.
9. *Show that when* $0 < |z| < 4$, $\frac{1}{4z-z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$
10. *Use Laurent series expansion to show that* $\int_C e^{\frac{1}{z}} dz = 2\pi i$ *where C is any positively oriented simple closed contour around origin.*
11. Find the residue at $z = 0$ of $f(z) = \frac{1}{z+z^2}$
12. Define pole of order 'm' and simple pole for a complex function $f(z)$

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Let f $f(z) = \begin{cases} \frac{\bar{z}^3}{z^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$

Prove that

a) f(z) is continuous everywhere on C

b) The complex derivative f'(0) does not exist

14. Verify CR Equations for the function $f(z) = (z^2 - 2)\, e^{-x}(\cos y - i \sin y)$

15. Prove that $|\exp(-2z)|<1$ if and only if $Re(z)>0$

16. Evaluate $\int_C \frac{z+2}{z}\, dz$, where C is the semicircle $z = 2e^{i\theta}, \ (\pi \leq \theta \leq 2\pi)$.

17. Prove that a function f is analytic at a given point, then its derivative of all orders are analytic at that point.

18. State and prove Liouville's Theorem.

19. *Suppose that if $z_n = x_n + iy_n$, $n = 1, 2, 3...$ and $z = x + iy$ Then show that $\lim_{n\to\infty} z_n = z$ if and only if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$*

20. State Cauchy's Residue Theorem. Using the theorem, evaluate $\int_C \frac{e^{-z}}{(z-1)^2} dz$, where C is the circle $|z| = 3$.

21. Define the improper integral of $f(x)$ over $-\infty < x < \infty$ and its Cauchy Principal Value. Show that the existence of Cauchy Principal Value does not imply the existence of $\int_{-\infty}^{\infty} f(x)dx$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that the function $u(x,y)=e^x(x \sin y + y \cos y)$ is harmonic and determine in terms of z, the most general analytic functon f(z) with real part u(x,y)

23. State and prove Cauchy's Integral Formula.
Find the value of $\int_C \frac{1}{(z^2+4)^2} dz$, where C is the circle $|z - i| = 2$ in the positive sense.

24. *a) Obtain a Maclaurin series expansion of $\frac{1}{1-z}$ when $|z|<1$. Deduce the expansions for $\frac{1}{1+z}$ about zero for $|z|<1$ and for $\frac{1}{z}$ when $|z-1|<1$*

b) Expand $f(z) = \frac{1+2z^2}{z^3+z^5}$ in powers of z and specify the domain in which the expansion is valid.

c) *Find a Maclaurin series expansion for* $f(z) = \sin(z^2)$ *and find out how it follows* $f^{(4n)}(0) = 0$ *and* $f^{(2n+1)}(0) = 0, n = 0, 1, 2, \ldots$

25. State and prove a necessary and sufficient condition for an isolated singular point z_0 of a function $f(z)$ to be a pole of order m. Derive the formula for residue at z_0 of $f(z)$. Find the residue at $z = i$ of $f(z) = \frac{(Logz)^3}{z^2+1}$.

(2×15=30)